UNIT 9

Debt Management





Debt management involves effectively handling various types of debt, such as credit card debt, college loans, personal loans and car loans. Managing debt effectively is crucial to avoid penalties and minimize interest payments.

Here are a few points to consider:

- Make sure to pay your debt obligations on time to avoid late payment penalties and additional fees. Set up automatic payments or create reminders to ensure you do not miss a due date.
- Focus on paying off high-interest debt first, such as credit card balances, as they tend to accrue more interest over time. By tackling these debts aggressively, you can save money on interest payments in the long run.
- Create a realistic budget that includes debt repayments. Allocate a portion of your income towards paying down debts systematically. By incorporating debt payments into your budget, you can manage your cash flow effectively and ensure you have enough funds to cover your obligations.
- Paying only the minimum amount due on your debts can result in a long repayment period and higher interest payments. Whenever possible, pay more than the minimum payment to accelerate debt repayment and minimize interest charges.

9.1 Credit Cards

Let's take a closer look at managing credit card debt through these math problems.

1 Samantha's credit card has a balance of $10,000. Her credit card company charges an annual interest rate of 17%. Calculate (a) the daily interest rate and (b) the balance on day two of her billing cycle.

2 Brian has an average daily balance of $1,200 on his credit card. The credit card company charges an annual interest rate of 16%. If there are 30 days in the billing cycle, how much will he be charged in interest during the current billing cycle?

3 Emily has a credit card balance of $3,500. If she makes monthly payments of $200, how many months will it take for Emily to pay off her debt?

4 Your credit card company requires you to pay 3% of your outstanding loan balance. You owe $6,000 on your credit card. Calculate your minimum payment.

5 Daniel has a credit card balance of $4,000. If he wants to pay off his debt in 36 months, what should his monthly payment be?

6 Sophia owes $2,800 on her credit card. If she wants to pay off her debt in 12 months, what should her monthly payment be?

7 You made a purchase of $200 on your credit card with an interest rate of 18% per year. If you pay off the entire balance in one month, how much interest will you pay?

8 You have a credit card with a balance of $500 and an interest rate of 20% per year. If you make a payment of $100, how much interest will you owe after one month?

9 Michelle's opening credit card balance is $1600 and she purchases $631 and repays $419. If the credit card company requires a minimum payment of 8% of the closing balance, find the minimum payment required.

10 Ibrahim has a credit card with a 19.5% annual interest rate and a 2% minimum monthly payment requirement. Ibrahim accumulates a balance of $3,150. What would be the minimum monthly payment at the end of the first month?

11 Sara has a credit card with an interest rate of 18% per year. If she has an outstanding balance of $500, how much interest will she be charged after 1 month?

12 Maya's credit card had a balance of $305.62 in June. She subsequently made a payment of $125 and purchased a bag for $290. Her credit card company charges an annual interest rate of 43.2%. Calculate the interest charge at the end of the month.

13 Jessica has a credit card with an annual interest rate of 18%. If her outstanding balance is $800, what will the interest charge be after two months if no prior payments are made?

14 Daniel has a credit card with an annual interest rate of 20%. If he has an outstanding balance of $2,500, (a) how much interest will he be charged after 2 months if he does not make any prior payments, and (b) what is the total amount he must pay if a 1% penalty fee is charged on the total balance after the second month?

15 Emily has a credit card with an outstanding balance of $2,500. The credit card company requires a minimum monthly payment of 3% of the outstanding balance or $50, whichever is higher. Calculate the minimum payment for this month.

16 Jason has a credit card with an interest rate of 24% per year. If he has an outstanding balance of $3,000, how much interest will he be charged after 3 months if no prior payments are made?

17 Sarah has a credit card with an annual interest rate of 16%. If her outstanding balance is $1,500, what will the interest charge be after 45 days if no prior payments are made?

18 Michael has a credit card with an opening balance of $2,650 and purchases $277 and repays $30. If the minimum payment required is 21% of the closing balance, find the minimum payment required.

19 Emma has a credit card balance of $3,560 and repays $300, what is her minimum monthly payment if the credit card company requires a 2% payment?

20 If Federer's credit card company charges an annual 6.5% interest rate, what is the daily interest rate?

21 Alexis purchases a handbag for $2,500 and a pair of boots for $600 on her credit card, which charges 18% annual interest rate. What is the total interest owed after 40 days if no prior payments are made?

22 Grace has a credit card with an annual interest rate of 14%. If her outstanding balance is $2,000, what is the daily interest on this balance?

23 Matthew has a credit card with an interest rate of 21% per year. If he has an outstanding balance of $3,000, how much will he owe including balance and interest after one month?

24 Sophia has a credit card with an annual interest rate of 18%. If her outstanding balance is $1,200, how much will she owe after 50 days?

25 Elon has a credit card with an annual interest rate of 42%. His credit card balance was 0 until the end of June when he purchased an item for $622. His monthly minimum payment is 9% of the balance.

- **A** Calculate his minimum payment for June.
- **B** Calculate the interest on the remaining balance.
- **C** Calculate the end of July balance (including interest incurred) after paying the June minimum payment.
- **D** Calculate the minimum payment for July.

26 Suppose that your credit card calculates interest using an annual interest rate of 44%. Your previous credit card statement had a balance of $340. So, you make a payment of $110. The next day, you use your credit card to purchase a new laptop for $1,215. Use this information to fill out the table below (including calculating the interest amount and new balance):

	Previous Balance	Payments	Purchases	Interest	New Balance
Month 1					

27 Continuing the same scenario from the previous question, assume that you make another payment of $85 and then use your credit card to purchase a mobile phone for $450. Us this information to complete the following table (including calculating the interest amount and new balance):

	Previous Balance	Payments	Purchases	Interest	New Balance
Month 1					

9.2 Loans

When borrowing or lending money, it is crucial to ensure clarity regarding whether the transaction is a loan or a gift. Here is a summary of important points to consider:

- Intent and Communication: Clearly communicate your intentions and expectations with the other party involved. Discuss whether the money is intended as a loan that must be repaid or as a gift with no expectation of repayment. Open and honest communication is essential to avoid misunderstandings and potential conflicts.
- Written Agreement: Regardless of whether it is a loan or a gift, consider documenting the transaction with a written agreement. A loan agreement should outline the loan amount, repayment terms (including interest, if applicable), and any other relevant conditions. For a gift, a written acknowledgment or gift letter can help clarify that the funds are not expected to be repaid.
- Terms and Conditions: For a loan, clearly define the terms and conditions, including the repayment schedule, interest rate (if any), late payment penalties, and any other relevant provisions. Both parties should have a clear understanding of their responsibilities and obligations.
- Legal Considerations: It is advisable to consult legal professionals to ensure compliance with applicable laws and regulations.

They can provide guidance on drafting loan agreements, gift documentation, and any legal implications or tax considerations associated with the transaction.

- Repayment Plan: If it is a loan, establish a repayment plan that suits both parties' financial circumstances. Determine the frequency and amount of payments, and document these details in the loan agreement. Regular communication regarding repayment progress can help maintain a positive relationship.
- Gift Tax Implications: If it is a gift, be aware of any potential gift tax implications. In some jurisdictions, gifts above a certain threshold may be subject to gift taxes. Consult tax professionals or relevant authorities to understand the applicable regulations in your specific situation.
- Financial Impact: Both borrowers and lenders should consider the potential financial impact of the transaction. Borrowers must evaluate their ability to meet repayment obligations, while lenders should assess the impact on their own financial situation and liquidity.

By ensuring clarity regarding whether a transaction is a loan or a gift and taking appropriate steps to document the agreement, establish repayment terms (if applicable), and address legal considerations, both parties can protect their interests and maintain a clear understanding of their financial obligations.

a. Simple Interest Loans

There are different formulas for calculating loan payments, depending on the type of loan and terms of repayment. Simple interest loans are commonly used for shorter-term loans or loans where interest is calculated based on a simplified method. Personal loans used for personal expenses or small business loans with shorter term, may be structured as simple interest loans.



To calculate the monthly payment on a simple interest loan, you can follow these steps:

Step 1: Calculate the interest accrued over the loan term
Interest = Principal x Rate x Time

Step 2: Calculate the total amount to be repaid
Total Amount = Principal + Interest

Step 3: Calculate the monthly payment
Monthly Payment = Total Amount/(time x 12)

Let's try an example:

1 You borrow $5,000 at an annual interest rate of 4.5% for a period of 2 years. Calculate (a) the total amount to be repaid and (b) the monthly payment.

SOLUTION

Step 1: Calculate the total interest accrued.

Principal x Rate x *Time Interest* = $5,000 *x* 0.045 x 2 = $450

Step 2: Calculate the total amount to be repaid.

Total Amount = *Principal* + *Interest*

Total Amount = $5,000 + $450 = $5,450

Step 3: Calculate the monthly payment.

Total Amount/(*Time x* 12)

Monthly Payment = $5,450/(2 x 12) = $227.08

Therefore, (a) the total amount to be repaid is $5,450, and (b) the monthly payment is $227.08.

Now, try the following problems.

2 You take out a loan of $10,000 with a 6% annual interest rate for 3.5 years (42 months). Find (a) the total interest paid and (b) the fixed monthly payment.

3 Principal: $8,500 Annual Interest Rate (Rate): 3.75% Time: 4 years. Find (a) the total amount to be repaid and (b) the fixed monthly payment.

4 Principal: $15,000 Rate: 5.25% Time: 2 1/2 years. Find (a) the total amount to be repaid and (b) the fixed monthly payment.

5 Principal: $12,500 Rate: 4% Time: 6 years. Find (a) the total amount to be repaid and (b) the monthly payment.

6 Principal: $20,000 Rate: 6.5% Time: 1 1/2 years (18 months). Find (a) the total amount to be repaid and (b) the monthly payment.

7 Principal: $6,000 Rate: 2.25% Time: 3 years. Calculate (a) the total amount to be repaid and (b) the monthly payment.

8 Principal: $9,500 Rate: 4.75% Time: 2 years. Calculate (a) the total amount to be repaid and (b) the monthly payment.

9 Principal: $25,000 Rate: 7% Time: 5 years. Calculate (a) the total amount to be repaid and (b) the monthly payment.

10 Principal: $7,500 Rate: 3.5% Time: 4 1/2 years. Calculate (a) the total amount to be repaid and (b) the monthly payment.

11 Principal: $18,000 Rate: 5% Time: 3 years. Find (a) the total amount to be repaid and (b) the monthly payment.

12 Principal: $10,500 Rate: 4.25% Time: 2.75 years (33 months). Find (a) the total amount to be repaid and (b) the monthly payment.

13 Principal: $14,000 Rate: 3.9% Time: 4.25 years (51 months). Find (a) the total amount to be repaid and (b) the monthly payment.

14 Principal: $8,000 Rate: 4.5% Time: 3.5 years (42 months). Find (a) the total amount to be repaid and (b) the monthly payment.

15 Principal: $11,200 Rate: 5.75% Time: 2.25 years (27 months). Find (a) the total amount to be repaid and (b) the monthly payment.



b. Compound Interest Loans

Use the following formula for calculating compound interest loans:

$$A = P(1 + r/n)^{(nt)}$$
$$\text{Interest} = A - P$$

1. You take out a loan of $10,000 with a 6% annual interest rate, compounded monthly, for 3.5 years (42 months). Find (a) the total interest to be paid and (b) the fixed monthly payment.
2. Principal: $8,500 Rate: 3.75%, compounded monthly Time: 4 years. Find (a) the total interest to be paid and (b) the fixed monthly payment.
3. Principal: $15,000 Rate: 5.25%, compounded monthly Time: 2 1/2 years. Find (a) the total interest to be paid and (b) the fixed monthly payment.
4. Principal: $12,500 Rate: 4%, compounded monthly Time: 6 years. Find (a) the total interest to be paid and (b) the monthly payment.
5. Principal: $20,000 Rate: 6.5%, compounded monthly Time: 1.5 years (18 months). Find (a) the total interest to be paid and (b) the monthly payment.
6. Principal: $6,000 Rate: 2.25%, compounded monthly Time: 3 years. Calculate (a) the total interest to be paid and (b) the monthly payment.
7. Principal: $9,500 Rate: 4.75%, compounded monthly Time: 2 years. Calculate (a) the total interest to be paid and (b) the monthly payment.
8. Principal: $25,000 Rate: 7%, compounded monthly Time: 5 years. Calculate (a) the total interest to be paid and (b) the monthly payment.
9. Principal: $7,500 Rate: 3.5%, compounded monthly Time: 4.5 years (54 months). Calculate (a) the total interest to be paid and (b) the monthly payment.

10 Principal: $18,000 Rate: 5%, compounded monthly Time: 3 years. Find (a) the total interest to be paid and (b) the monthly payment.

11 Principal: $10,500 Rate: 4.25%, compounded monthly Time: 2.75 years (33 months). Find (a) the total amount to be repaid and (b) the monthly payment.

Let's Explore

12 Let's compare two loan options: a simple interest loan and a compound interest loan. Consider a $10,000 loan at 5% interest repaid over five years.

- **A** Calculate the cost of borrowing (principal + interest) if it is a simple interest loan.
- **B** Calculate the cost of borrowing (principal + interest) if interest is compounded monthly.
- **C** Compare the results of (a) and (b) and think about other factors to consider when making a decision to take out a loan.

c. Amortized Loan Formula

The following amortization formula is used for loans with equal monthly payments that include both principal and interest. The formula calculates the payment amount based on the loan amount, interest rate and loan term.

$$M = P \times (r \times (1 + r)^n) / ((1 + r)^n - 1)$$

Let's break down the formula used to calculate the monthly payment on a loan:

M: Monthly payment
P: Loan principal (amount borrowed)
r: Monthly interest rate
n: Total number of payments



The formula is derived from the concept of an amortizing loan, where each monthly payment consists of both principal (the amount borrowed) and interest (the cost of borrowing). The goal is to distribute the repayment of the loan over a fixed number of monthly payments.

Here's how the formula works step by step:

- Step 1: Convert the annual interest rate to a monthly interest rate. To calculate the monthly interest rate (r), you divide the annual interest rate by 12 and convert it to a decimal. For example, if the annual interest rate is 6%, the monthly interest rate would be 0.06/12 = 0.005.
- Step 2: Determine the total number of payments. Multiply the number of years (n) by 12 to get the total number of payments. For example, for a 30-year loan, the total number of payments would be 30 x 12 = 360.
- Step 3: Calculate the numerator. Multiply the loan principal (P) by the monthly interest rate (r) and then multiply it by (1 + r) raised to the power of the total number of payments (n).
- Step 4: Calculate the denominator. Calculate (1 + r) raised to the power of the total number of payments (n) and subtract 1.
- Step 5: Divide the numerator by the denominator and multiply the result by P, to get the monthly payment (M).

It's important to note that this monthly payment calculation assumes fixed interest rates and equal monthly payments throughout the loan term. Additional fees or changes in interest rates may affect the actual loan payments.

The first one has been solved for you.

1 Megan wants to finance a car and is considering a loan of $25,000. The bank offers her an interest rate of 4.5% per year for a term of 5 years. Calculate Megan's monthly car loan payment.

SOLUTION

Step 1: Convert the annual interest rate to a monthly interest rate.

Annual interest rate = 4.5% = 0.045 (*as a decimal*)

Monthly interest rate = 0.045/12 = 0.00375

Step 2: Convert the loan term to months.

Loan term (n) = 5 *years* × 12 *months/year* = 60 *months*

Step 3: Use the amortization formula to calculate the monthly payment.

Principal loan amount (P) = $25,000

Monthly Payment = $P \times (r(1+r)^n) / ((1+r)^n - 1)$

Plugging in the values:

$25{,}000 \times (0.00375(1+0.00375)^{60}) / ((1+0.00375)^{60} - 1)$

Step 4: Simplify the equation.

Calculate $(1+0.00375)^{60}$ and $((1+0.00375)^{60} - 1)$ separately.

$(1+0.00375)^{60} = 1.251795821$

$(1+0.00375)^{60} - 1 = 0.251795821$

Monthly Payment = 25,000 × (0.00375 × 1.251795821)/0.251795821

Step 5: Calculate the monthly payment.

Monthly Payment = $466.08 (*rounded to two decimal places*)

Therefore, Megan's monthly car loan payment for a $25,000 loan at an interest rate of 4.5% for 5 years is $466.08.

2 Michael wants to purchase a car and decides to take a loan of $20,000. The bank offers him a loan with an annual interest rate of 6.25% for a term of 4 years. Calculate Michael's monthly car loan payment.

3 You decide to get a car with a loan. *Car loan Amount*: $15,000 *Annual Interest Rate (Rate):* 5% *Loan Term*: 3 years. What is your monthly payment?

4 Calculate the monthly payment: *Car loan Amount:* $22,500 *Rate:* 6.75% *Loan Term:* 5 years.

5 Calculate the monthly payment: *Car loan Amount:* $12,000 *Rate:* 4.25% *Loan Term:* 2 years.

6 *Car loan amount:* $18,500 *Rate:* 5.75% *Loan Term:* 4 years. Calculate the monthly payment.

7 *Car loan amount:* $30,000 *Rate:* 3.75% *Loan Term:* 6 years. Calculate the monthly payment.

8 *Car loan amount:* $25,500 *Rate:* 4.8% *Loan Term:* 5 years. Calculate the monthly payment.

9 Amy took out a student loan. *Loan Amount:* $14,000 *Rate:* 6.5% Loan Term: 3 years. Calculate the monthly payment

10 Ibrahim took out a student loan. *Loan Amount:* $32,500 *Rate:* 4.2% *Loan Term:* 4 years. Calculate his monthly payment.

11 Johanna decides to take out a student loan. *Loan Amount:* $9,800 *Rate:* 5.25% *Loan Term:* 2 years. Calculate her monthly payment.

12 You decide to take out a student loan. *Loan Amount:* $22,000 *Rate:* 6.1% *Loan Term:* 5 years. Calculate your monthly payment.

13 Aisha needs to get a student loan. *Loan Amount:* $18,500 *Rate*: 4.9% *Loan Term:* 4 years. Calculate her monthly payment.

14 I am thinking of getting a student loan. *Loan Amount:* $12,700 *Rate:* 5.75% *Loan Term:* 3 years. What will my monthly payment be?

15 *Car Loan Amount:* $21,000 *Rate:* 4.25% *Loan Term:* 5 years. Calculate the monthly payment.

16 *Student Loan Amount:* $15,200 *Rate:* 3.9% *Loan Term:* 4 years. Calculate the monthly payment.

17 *Car Loan Amount:* $10,600 *Rate:* 4.6% *Loan Term:* 3 years. Calculate the monthly payment.

18 *Student Loan Amount:* $15,000 *Rate:* 6.2% *Loan Term:* 5 years. Calculate the monthly payment.

19 *Student Loan Amount:* $25,000 *Rate:* 4.8% *Loan Term:* 7 years. Calculate the monthly payment.

20 *Student Loan Amount:* $12,500 *Rate:* 3.5% *Loan Term:* 3 years. Calculate the (a) monthly payment and (b) total interest paid on entire loan.

21 *Student Loan Amount:* $9,000 *Rate:* 4.2% *Loan Term:* 4 years. Calculate the (a) monthly payment and the (b) cost of borrowing (principal + interest) by end of term.

22 *Student Loan Amount:* $22,500 *Rate:* 5.1% *Loan Term:* 6 years. Calculate the (a) monthly payment and (b) total interest paid on entire loan.

(i) Amortization Schedule

An amortization schedule is a detailed table that outlines the repayment of a loan over time. It breaks down each payment into its principal and interest components, showing how the loan balance decreases as payments are made. Amortization schedules are commonly used for mortgages, car loans, and other installment loans where regular payments are made over the course of the loan term.

The key components of an amortization schedule include:

Payment Number: The sequential number of the payment.

Payment Amount: The total payment amount that needs to be made in each period, including principal and interest portions.



Principal Payment: The portion of the payment that goes towards reducing the loan's principal amount.

Interest Payment: The portion of the payment that covers the interest charged on the remaining loan balance.

Total payment: The sum of the principal payment and the interest payment.

Remaining Principal/Balance: The outstanding loan balance after each payment is made. It decreases as the loan is paid off.

By the end of the loan term, all principal and interest should be paid off, ending with a zero balance.

Let's create an amortization schedule based on the following scenario:

You want to get a loan for $100,000. The bank offers you an interest rate of 1.79% per year for a term of 12 months. Create an amortization schedule for your payments, assuming fixed monthly payments.

Calculate the monthly payment using the amortization formula.